Optimizing Adaptation of Smart Traffic Lights with Resource Constraints

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Abstract—Large cities worldwide are facing many challenges, such as a growing population, which leads to an increase in traffic congestion. In many places, city road infrastructure does not cope well with growth in the number of vehicles leading to long waiting times on roads and decreasing fuel economy. Due to inflexibility to update city roads comfortably in most places, the adaptation of smart traffic intersections provides a promising approach to curb growing traffic and reduce wait times on road intersections. Still, faltering public budgets do not allow costly transport infrastructure expenditures, and deploying and maintaining sensors to enable smart intersections throughout a city is resource-intensive.

In this paper, we explore the idea of efficiently deploying smart intersections within given budget constraints in a city. We propose a generic simulation-based framework that models city traffic based on historical patterns and utilizes agent-based modeling to select an optimal subset of road intersections to improve traffic conditions for a given city network. We validate our framework with state of the art urban mobility framework and demonstrate its utility with a simple road network. We further employ our framework to study the impact of variation in traffic characteristics and budget constraints on the efficiency of deploying smart lights.

Index Terms—Urban Planning, City Traffic Simulation, Agent Based Modeling, Smart Traffic Management Systems, Internet of Things

I. INTRODUCTION

Traffic congestion is one of the major problems in many major metropolitan cities globally and leads to significant concerns like transportation delays, increase in pollution level, and unnecessary fuel consumption. As the number of vehicles increases, prior research investigates methods to modify traffic demand and density to alleviate traffic problems in congested areas [1]–[3]. One of the most critical problems in cities is the difficulty of effectively expanding the existing infrastructures, if it is even feasible. A promising approach is to provide better management of the traffic lights using technology. Traffic light signals at intersections and traffic management systems play a vital role in effectively controlling traffic. The conventional pre-timed controlled traffic signals are not ideal in dealing with the intense congestion levels. Especially during rush hours, i.e., heavy traffic at one of the roads on an intersection could be waiting unnecessarily while there is no rush on other roads whose signal is green. Thus, much work in this area proposes the idea of smart traffic lights, which automatically adjusts their timings based on traffic density on a given intersection [4]-[8].

These works show success in controlling traffic congestion by prototyping smart light models [4], [9]-[11]. There are numerous efforts in integrating these smart solutions to end-toend traffic control systems [12]-[17]. Researchers also looked at reinforcement learning-based systems that automatically adapt to traffic conditions [18]. Still, most of the big cities around the world do not adopt these smart traffic lights. Two key reasons for their reluctance are (i) Lack of a universally accepted method to design intelligent traffic lights, since they are dependent on traffic characteristics and road network topology, (ii) deployment and maintenance of smart lights across the city is resource-intensive. There are various methods available to detect traffic congestion levels, such as video data analysis, infrared sensors, inductive loop detection, wireless sensor networks, and few other technologies [19]. Each of these methods needs setting up and looking after sensors across the city, taking serious efforts from deployment and long-term maintenance perspective. Typically, cities have a different budget for promoting smart lights, and in most cases, not enough to deploy and maintain sensors around the whole city.

In this paper, we developed a simulation-based framework to facilitate smart traffic light adaptation for different kinds of road network topology, traffic characteristics, and signal control mechanism under given resource constraints. Our primary motivation is to design a generic system, which can be utilized by various stakeholders across cities for deploying smart lights in an intelligent manner given their resource constraints and can provide optimal utility in terms of improving traffic conditions. Our framework utilizes agent-based modeling techniques to model city traffic. Table I shows a comparison of our models with similar traffic simulation models available in the literature. We describe a method to model city traffic based on historical traffic patterns (Section II). Further, we propose a greedy approach to iteratively select a subset of intersections in the city to convert into smart intersections, which leads to the most impact on alleviating traffic congestion and improve average vehicle movement speed across the city (Section III). Being an iterative approach, this gives flexibility to city management to optimize selection for given budget constraints.

We validate our model by comparing traffic throughput for a given topology and traffic conditions with a well-known urban mobility simulation (SUMO) framework (Section IV). We show our approach's effectiveness compared to random selection by modeling a small-scale road network, traffic characteristics supported by real data, and traffic density based smart lights (Section V). Further, we study the impact of traffic characteristics like traffic density, the speed limit of cars, and budget constraints on overall traffic congestion reduction with the given approach (Section VI).

TABLE I: Comparing our model with existing agent based models for city traffic simulation.

	Atismart [20]	SUMO [21]	Our Model	
Parameters				
Macro Traffic Characteristics		*	*	
Road Topology Network	*	*	*	
Multi-Road Junctions	*	*	*	
Key Variables				
Cars, Traffic lights	★	*	*	
Multiple Types of cars		*		
Pedestrians, Multiple Lanes		*		
Different Street types		★	*	
Smart (Traffic Density			*	
based) Lights				
Processes				
Collision Free Movement	★	*	*	
Navigation Capabilities	★	★	★	
Arrival/Departure of Vehicles	★	★	★	
Changing Road Lanes		★		
Acceleration, Speed Control		*	*	
Towing	★			
Layout Independent Simulation			★	
Re-configuring Traffic Control			*	
System				

II. CITY TRAFFIC MODELING

We can model traffic for any place as the movement of vehicles through the network. Each vehicle is modeled using their start time, type of vehicle, start location, destination, and path they follow on the network. An absolute knowledge about traffic conditions would be the ability to model all vehicles for a given timeframe. However, predicting exact traffic conditions for any network is improbable. Even when traffic around a network is unpredictable, it is not random. Some key patterns broadly define how traffic on roads would be like for a given day.

Overall Traffic Density: The number of people who commute on an average over a given day and the variability observed across different days.

Time of Day Variation: Time of day is another crucial aspect of modeling traffic. In general, overall vehicle density during night times is low, and we observe peak traffic conditions during the morning and late evening hours when people commute for work.

Day of Week Variation: Like most people in the city commute for work purposes, there can be significant differences between weekend and weekday traffic patterns, which attributes to traffic variation across days.

Location based Variation: Traffic conditions and overall flow of traffic can vary in any city (network) based on what kind of areas we are looking in. For a given city, most people will be commuting from residential areas to commercial areas during morning hours and commercial areas to residential areas during evening hours [22]. Also, more people will be originating from high population density areas in comparison to other regions.

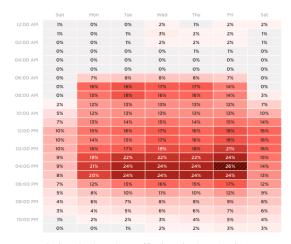
We utilized these factors to seed traffic modeling in terms of the movement of vehicles on roads. This information for any city is available based on traffic reports [23], classification of sub-areas into residential/commercial, and population density variation across the city. For example, Figure 1a shows the average weekly variation in traffic for the city of Pittsburgh. This information together defines traffic characteristics for a given network.

To generate vehicle movement for a given day, we start with modeling the number of vehicles running that day using a normal distribution with traffic density information as parameters. To incorporate the hour of day variation, we sample the start hour for each vehicle from a multinomial distribution across 24 variables, one variable for each hour. Each hour's probability is based on hourly traffic density, and we sample start minutes from a uniform distribution. Based on variation in population density and type of areas, we assign each intersection start and end probability of cars for a given hour and further sample source and destination using multinomial distribution across all intersections (nodes) in the road network. For simplicity, we assume vehicles start and end from intersections closest to their source and destination, respectively. Algorithm 1 details steps for formally modeling traffic. Modeling various kinds of vehicles is currently out of scope for this paper but can be easily extended by modeling different sizes and movement speeds for all vehicles.

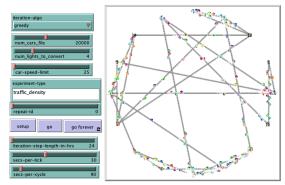
III. SIMULATION DESIGN

Our simulation framework is implemented in Netlogo. It is a multiagent programming language and modeling environment for simulating complex phenomena. Cars and traffic Intersections are modeled as turtle agents, and roads are modeled as link agents (see Figure 1b). Our simulation model's primary inputs are road network topology given as a directed graph and traffic information provided as vehicles, their start time, source, and destination. We generate traffic information separately using Python 3.8, which takes input traffic characteristics and resource constraints in terms of the maximum number of traffic intersections we can convert to smart intersections. There are other configurable parameters like (i) speed limit, which determines top speed cars can have, (ii) seconds per tick, which controls how many seconds does one tick simulate in one round, (iii) seconds per cycle, which represents the phase of a conventional pre-timed signal when it switches between the stop and go state, (iv) iterationstep-length, which represents how many hours after which we decide to convert an intersection into a smart intersection, and (v) acceleration, which represents acceleration/deceleration of cars in m/s^2 .

Networks created in NetLogo (Figure 1b) do not follow the exact layout of physical road topology. Without information



(a) Variation in hourly traffic(in Pittsburgh city) across hour of the day and day of the week



(b) Netlogo Interface: configurable parameters, and visualization for city traffic simulation

Fig. 1: Traffic Characteristics and Simulation Design

about exact geographical coordinates for all road intersections, representing exact road topology in a visualization window is a challenging problem of planar graph representation [24]. For the purpose of our simulation, exact coordinate information is not required as we use actual road lengths (which is easier to acquire) and convert vehicles' speed from m/s to units/ticks at each tick, utilizing the ratio of actual road length(in meters) and apparent road length on visualization window(in pixels) in the NetLogo network. Cars move on a road network from the start node(intersection closest to source) to the end node(intersection closest to destination) following Dijkstra's shortest path on directed graphs [25]. Vehicles can change their route based on traffic conditions in the real world, but it is out of this simulation's scope. We assume vehicles do not modify the route in real-time based on traffic conditions on the network.

To determine a subset of intersections where we should deploy smart lights to minimize overall congestion, we use an iterative process. We simulate one iteration step length (set to one day), record insights on traffic conditions for this simulation step, and use it to decide which node is a bottleneck for traffic congestion based on a greedy logic and make that node smart. There can be many different methods to choose

Algorithm 1: Model City Traffic

num-days (number of days to run in simulation); Avg-cars (average traffic in terms of cars per day); var-cars (deviation in traffic in terms of cars per day); hourly-var (hourly variation in traffic for each hour); node-src-var(h) (Source probabilities in given hour h for each node); node-dest-var(h) (Destination probabilities in given *hour h for each node*) : cars-info (Empty Array to store resulting vehicle *movement information*) : for $day \leftarrow 0$ to num-days do num-cars←get one sample from NormalDist(Avg-cars, var-cars) cars-start-hours \leftarrow get num-cars samples from *Multinomial*(hourly-var) cars-start-mins \leftarrow get num-cars samples from UniformDist(1,60)cars-src, cars-dest \leftarrow Empty arrays for size num-cars for $hour \leftarrow 0$ to 24 do for indexes cars-start-hours = hour do cars-src[*indexes*] \leftarrow get num-indexes samples from *Multinomial*(node-src-var(hour)) cars-dest[*indexes*] \leftarrow get num-indexes samples from Multinomial(node-dest-var(hour)) end end cars-info.append(zip(day,cars-start-hours,cars-startmins,cars-src,cars-dest)) end Result: cars-info

bottleneck nodes for any iteration step. A few of them can be: (i) Choose nodes with the maximum average waiting time or maximum count of vehicles for complete iteration step or (ii) Choose nodes with the maximum difference in average waiting time or the number of vehicles on the incoming roads to that node. Furthermore, various methods can make a traffic light smart, like changing the phase of traffic lights, i.e., increasing the duration of green lights for incoming roads with a higher density of traffic, or redistributing the time allotted for one complete cycle on traffic density on incoming roads. There is no single method that works best for all kinds of network topologies and traffic characteristics. Thus, we made the selection of the greedy algorithm and approach to smart lights configurable in our framework. After converting our bottleneck node into a smart node, we continue the simulation with this new configuration in the next step of simulation and repeat the process till we exhaust our budget to convert smart lights.

IV. MODEL VALIDATION

To validate the inner workings of our model, we utilize the model to model comparison with SUMO (Simulation of Urban Mobility) [21] model. It is an open-source, portable, microscopic, and continuous multi-modal traffic simulation package designed to handle large networks. It is developed and validated by the German Aerospace Center. It is used for research purposes like traffic forecasting, evaluation of traffic lights, route selection, or vehicular communication systems. It supports many features like pedestrians, multiple road lanes, different vehicle types, and complicated road and intersection topologies. However, there are many limitations to their modeling approach compared to our model. It requires exact location-based input for intersections, does not allow reconfiguration of traffic control systems during simulation, and does not support intelligent traffic management systems based on traffic density. Therefore, our work is meant to improve upon previous work's foundational concepts.

We design validation run on a sample network with ten intersections with pre-generated traffic for the length of 5 days. We generate configuration files for SUMO and remove all additional features. Next, we run both SUMO and our NetLogo model for the same input. As SUMO does not allow for density-based smart lights, we restrict using pretime traffic lights for this NetLogo simulation. Our t-test on the distribution of unique vehicle counts on an hourly basis on the network results in a p-value of 0.86. Figure 2 shows the comparison of traffic density over time on a complete network for SUMO and NetLogo simulations. We also observe that unique vehicles count on a given road over time are similar for both models, with a median p-value around 0.95 across all roads. This analysis shows that the distribution for overall traffic density is similar for both models. The similarity between traffic levels based on this experiment validates that the NetLogo model behaves as expected and close enough to emulate real-life traffic behavior.

V. EXPERIMENT DESIGN

Our experiment design's motivation is to show the efficacy of our modeling approach and present how we can assess impacts of traffic characteristics, budget constraints, and overall network topology on alleviating traffic congestion with smart lights with our model. We run our experiment on a sample road network with 16 nodes (intersections) and 25 roads. As intersections with more than four incoming roads are rare, all nodes in our network have degrees less than 5. Traffic characteristics like average traffic density, variation across days, and hourly variation are modeled based on traffic data collected from the Pittsburgh downtown area [26]. As any real city network does not inspire our sample road network, we randomly set location-based variation on the road network. To identify bottleneck nodes as a part of our greedy logic in each iteration, we select the worst traffic nodes with a maximum average number of vehicles waiting during the busiest hour of the day across all non-smart intersections during our iterations. For smart traffic lights, we use a traffic

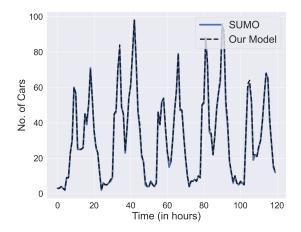


Fig. 2: Model to Model Comparison: Validating inner working of our model based on similarity in hourly traffic density for SUMO and Netlogo simulation over time

density based approach, i.e., after each traffic-signal cycle, each node assesses traffic from all incoming roads and turns green for roads with maximum cars waiting on the signal.

Every iteration in our simulation is a day long. With each iteration, we select an intersection to convert based on the previous iteration's traffic conditions and continue this process till we exhaust our budget. The average run time per iteration for any simulation is dependent on configured parameters like the complexity of road topology, traffic density, and length of simulation for each iteration. For our experiments, it took us on an average between 1-5 minutes per iteration based on the traffic density of our simulation. For the sake of generalizability, we quantify the budget in terms of the maximum number of smart lights we can convert. We compare improvements over time of our greedy approach with a random selection of traffic intersections. The count of intersections selected randomly for each iteration is kept the same as that with the greedy approach to maintain fair comparison. We also assess improvements from baseline (conventional traffic lights) conditions. We observe changes in traffic congestion over time for a complete network based on these factors, (i) Average queue length (per hour) of cars waiting on most busy roads in the network(see Figure 3a) (ii) Distribution of average speed of cars simulated on the network (see Figure 3b), and (iii) the average number of cars waiting on bottleneck(worst traffic) nodes during peak hours(see Figure 3c).

Further, We assess smart traffic lights' impact with changes in traffic density and movement speed of vehicles in the city. We also analyze the impact of having different budget constraints for smart lights. Each experiment is repeated five times, and impact over traffic conditions is averaged over all repetitions. Table II summarises our virtual experiments, and our findings are presented in the following section.

TABLE II: Virtual Experiment Table

Items	Values	Count		
Independent Variables				
Traffic Density	5000,10000,20000,40000	4		
Traffic Light Logic	None, Random, Greedy	3		
Budget(Count of Traffic Lights)	0,4,8,12,16	5		
Control Variables				
Road Topology	16 Nodes, 25 Roads	-		
Seconds-per-cycle	90	-		
Seconds-per-ticks	30	-		
Speed Limit	25 m/s	-		
Output Variables				
Speed of Cars	0- speed limit -			
Wait time of Cars	0- ∞	-		

^aEach experiment repeated 5 times. This is a 4-3-5-5 design.

VI. RESULT ANALYSIS

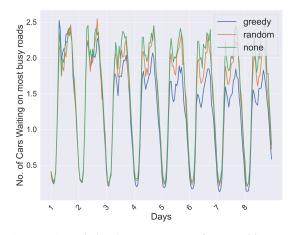
We observe that having more smart lights leads to an overall decrease in traffic congestion in terms of the number of vehicles waiting over busy nodes. We also observe that our greedy approach fares better than random selection, and the difference keeps increasing as the number of smart intersections increases (Figure 3a). Also, as the number of smart lights increases, overall speed distribution for the same traffic shifts more and more towards the right, which suggests an increase in the average speed of cars, leading to more driving comfort (Figure 3b). We also observe a significant decrease in the average number of cars waiting at the worst traffic intersection in iteration over time (Figure 3c). In summary, opting for a greedy approach with our model leads to improvement in traffic conditions compared to random selection. Both approaches introducing smart intersections improve traffic conditions compared to the conventional traffic light approach.

1) Impact with change in traffic density: As overall traffic density increases in terms of the average number of people commuting in a day, the greedy approach becomes more effective than the random selection of nodes. However, there is more than a 50% improvement in average no. of cars waiting at most busy intersections compared to conventional traffic lights in both high and low traffic density conditions with a greedy approach (Table III). We also observe that improvements in cars' average speed (i.e., Driving comfort) are agnostic to traffic density in the city over time. The scale of improvement in terms of worst node traffic also increases with an increase in traffic density; however, it is important to note that the scope for improvement is also significantly higher with high traffic density.

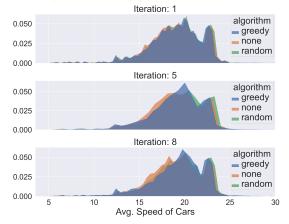
TABLE III: Improvement with change in Traffic Density(with50% Budget)

Traffic Density	%Improvement in traffic conditions over conventional traffic lights		
(Average #cars per day)	Random Approach	Greedy Approach	
5000	0.42%	59.27%	
10000	19.55%	65.39%	
20000	15.65%	54.75%	
40000	35.48%	96.33%	

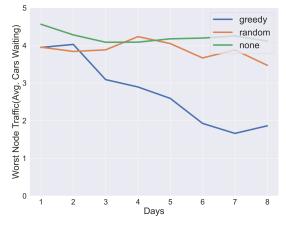
2) Impact with change in budget constraints: Traffic during peak times decreases more with more budget to make traffic



(a) Hourly variation in average No. of cars waiting on most busy roads over days



(b) Distribution of average speed of all cars that ran on the network for first, middle and last day of simulation



(c) Average No. of cars waiting on bottleneck(worst traffic) nodes during peak hours for over days

Fig. 3: Traffic Characteristics Over Time

lights smart. Also, as the number of smart lights increases, returns in terms of improvement in traffic congestion diminishes (Table IV). More experiments with higher numbers of nodes can provide a clear picture of diminishing returns. Also, there is an improvement in the overall distribution of average speed as the smart lights' budget increases.

TABLE IV: Improvement with change in Budget Constraints (with 20000 cars per day)

Budget	%Improvement in traffic conditions over conventional traffic lights		
% Lights	Random Approach	Greedy Approach	
25%	12.03%	36.16%	
50%	13.19%	66.45%	
75%	25.71%	94.15%	
100%	60.32%	100.00%	

In summary, we conclude that having smart lights on the city network generally helps decrease traffic congestion. Areas with higher traffic density require more attention (greedy approach) to select which intersections should be made smart. Also, cities that do not have a large budget can still significantly improve traffic conditions by making a small fraction of traffic lights smart using our greedy approach. I.e., for the road topology we use in experiments, making 25% of specific traffic lights smart leads to around 36% improvement in overall traffic conditions over conventional traffic lights.

Based on overall simulation design, our experiments can be extended further in multiple directions. We can explore impact of introducing different kinds of vehicles on roads based on characteristics like speed limit and acceleration. We can also assess improvements in traffic conditions over different kinds of network topologies, and how our greedy approach for smart node selection fares with different levels of sparsity in road networks.

VII. CONCLUSION

This paper proposes a new simulation-based framework to optimize smart lights adaptation within budget constraints for any given road network and traffic characteristics. We show our modeling approach's efficacy with a simple road network and traffic characteristics based on real data in selecting optimum intersections to deploy traffic density driven smart lights. We also assess the impacts of variation in traffic density, movement speed, and budget constraints for a given network topology. Given the generalizability of the framework, we can utilize it for various kinds of experiments like studying the impact of different types of network topologies, different types of vehicles based on movement speed, different kinds of greedy approaches, and various mechanisms deploying smart lights on intersections. Further, our simulation interface is scalable to run simulations on much larger networks for long periods for sustained analysis. We can also incorporate it as part of larger Urban Mobility Models like SUMO [21] moving forward.

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